

Experimental Rapid Surface Heating by Induction for Micro-Injection Molding of Light-Guided Plates

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Received 10 May 2008; accepted 5 January 2009

DOI 10.1002/app.30053

Published online 8 April 2009 in Wiley InterScience (www.interscience.wiley.com).

ABSTRACT: This work experimentally investigates the use of induction-heating to heat mold surfaces rapidly, and thus enhance the replication effect of the microstructure of light-guided plates (LGP) in the injection molding process. This investigation employs a 2-inch LGP injection mold as the experimental carrier, and compares the replication effect on the microstructure of induction heating with that of conventional oil-heating. Temperature increases on the mold plate are examined using a thermal video system. The experimental

results show that (1) the flat induction coil design promotes rapid surface heating. (2) Induction-heating the mold surface to 110°C improves the replication rate of the height of the micro-structure by up to 95%. (3) The LGP produced by induction heating has no significant residual stress. © 2009 Wiley Periodicals, Inc. *J Appl Polym Sci* 113: 1345–1354, 2009

Key words: induction heating; injection molding; light-guided plate; microstructure; rapid surface heating

INTRODUCTION

The light-guided plate (LGP) is an important component of a back light unit that affects the optical efficiency and is mainly used to guide scattered rays to improve the uniformity of brightness of the panel. However, whether the ray may travel in the desired direction depends on the replication effect of the LGP's microstructure during injection molding. For instance, if the mold temperature is too low, especially for microstructure with a high aspect ratio and a micrometer-scale width, the melt plastics may be frozen before the mold is completely filled. Hence, a suitable mold temperature control system must be adopted. A high mold temperature may reduce the number of defective finished products and increase the likelihood of complete filling of the microstructure with high aspect ratio.^{1,2} However, traditional methods, such as oil-heating and electric heating cartridge, for heating the mold have a very low rate of heat transfer, and melt plastics must be cooled into the solid state to form the finished products. As a result, the mold temperature is high and the cooling time increases, increasing the cycle time and, thereby, the production cost.

A number of studies suggested that a high mold temperature promotes feature transfer in micro

molding. Given the quality demanded of various products and the cost benefits, methods of surface rapid heating have been investigated.^{3–6} They are mainly flame heating, coating heating, infrared thermal radiation heating and induction heating. Table I presents the advantages and disadvantages of the main current heating techniques and their principles of operation. Induction heating is used for its skin effect, by which heat transfer occurs only on the surface of the mold cavity that is heated by the induction coil. Accordingly, the heat absorption efficiency of the surface of the cavity is high and the heat on the surface is rapidly dissipated. Moreover, the control of power, heating distance, and design of the coil directly govern heat transfer efficiency. Because induction heating is non-contact heating, it does not change the intrinsic structure of the mold. The heating efficiency of induction heating relies on heated target's material properties and the oscillating frequency. In the past studies,⁷ induction heating can heat the mold surface rapidly over 20°C per second. As heating coils may be replaced according to the shape or depth of the cavity, induction heating can be used to heat various complex mold surfaces.

In this work, induction heating is adopted to provide the high mold temperature that is required for the short filling stage during the replication of LGP's microstructure, and also to reduce rapidly the mold temperature during cooling to reduce further the cooling time of the molded parts. As the mold temperature is an important injection molding parameter that affects the molecular orientation, residual

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TABLE I
A List of Recent Mold Surface Rapid Heating Methods

	Flame heating ³	Coating heating ⁴	Infrared ray heating ⁵	Induction heating ⁶
Operation principle	<ul style="list-style-type: none"> • Heating with gas flame to raise temperature of the mold surface instantly 	<ul style="list-style-type: none"> • One thin metal layer and one thermal insulation layer are coated on the mold cavity, and the thin metal layer is heated with copper poles to achieve the purpose of controlling temperature. 	<ul style="list-style-type: none"> • Infrared ray illuminants are produced with halide lamps and baffle-boards behind to heat the workpiece rapidly. 	<ul style="list-style-type: none"> • Input high frequency alternating current in the induction coil, and heat the surface of the workpiece with the induction current produced on it.
Advantages	<ul style="list-style-type: none"> • Heating rapidly 	<ul style="list-style-type: none"> • Being sensitive • High utilization ratio of energy • Temperature is easy to be controlled 	<ul style="list-style-type: none"> • Mold surface may be heated instantly • Non-touch heating • Mold structure is not required to be changed 	<ul style="list-style-type: none"> • Mold surface may be heated instantly • Non-touch heating • The intrinsic structure of the mold is not required to be changed. • Heating coils may be changed according to mold cavities of different shapes, and it is applicable for heating complicated mold surfaces.
Disadvantages	<ul style="list-style-type: none"> • A cooling water circuit similar to that of the traditional mold is required to transport fuel and gas. Its design is complicated. • Problems regarding safety 	<ul style="list-style-type: none"> • It is difficult to film the section of complicated and tiny mold cavity. • Hidden trouble of safety exists in insulation of the resistor layer. • The life time of resistor heating layer is limited. 	<ul style="list-style-type: none"> • High construction cost • As it is in irradiation form and limited by projection area, it is difficult to heat lengthways long or complicated mold cavities uniformly. 	<ul style="list-style-type: none"> • High initial construction cost. • The coil is consumable and must be specially designed. • In operation field, electronic instruments around will be affected.

stress, shrinkage and warpage of molded parts, the effect of mold temperature on the quality of the molded parts is further discussed.

PRINCIPLES OF RAPID INDUCTION HEATING

Induction heating involves heating metal by electromagnetic induction. According to Faraday's law and Lenz's law, passing electrical AC through heating coils produces an alternating magnetic field that can be used to heat an object. If processed magnetic or non-magnetic conductive workpieces are placed in the alternating magnetic field established by the heating coils, the cutting-of-flux causes a current (I_c) to be produced at different depths. The resistance of workpieces and the flow of eddy currents therein generate heating power of $I_c^2 R$.⁸

Induction heating exploits two effects—the Joule effect and the electromagnetic effect. The Joule effect

is based on hysteresis loss and eddy current loss, which are defined as follow.

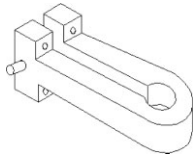
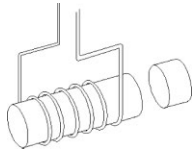
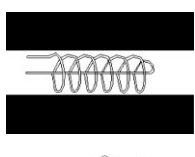
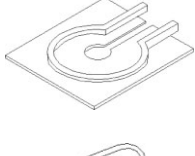
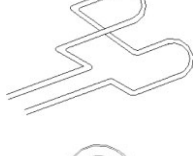
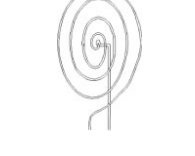
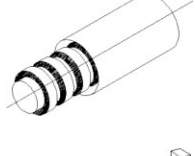
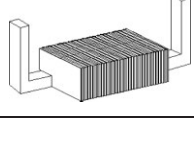
Hysteresis loss

Hysteresis loss is related to non-contact electromagnetic induction between the heating coils and the workpieces. As the workpieces undergo magnetization, de-magnetization and re-magnetization, the hysteresis loss, which results mainly from ferromagnetic material, is converted into heat. The hysteresis loss is as follows⁹;

$$P_h = K_h \times f \times B_m^x \times U, \quad (1)$$

where P_h denotes the hysteresis loss of the processed workpiece; K_h is hysteresis constant; f is current frequency; B_m is the peak flux density; x is a material coefficient, and U is the volume of the processed workpiece.

TABLE II
A List of Various Types of Induction coils and their applications

Shape	Figure	Material	Features and purpose
Single-turn		Metal and insulation material	Applicable for hardening and local heating requirements of column shaped load
Multiturn coil shape		Copper	Number of turns is reduced and increased flexibly according to the heating length of the load.
Interior screwy shape		Copper	Degree of heating may be increased by adding proper amount of core materials, and it is applicable for complicated load generally.
Single turn plane shape		Thin copper sheet	Applicable for heating load of narrow scope
Single-turn bobby pin shape		Copper	Applicable for heating different parts
Single-turn flat shape		Copper	Applicable for heating plane.
Multi-level		Copper	Applicable for low current and high voltage heating
Core inductance type		Ferronickel	More applicable for quenching of edge angle shaped load sheet

Eddy current loss

When an alternating current is passed through the heating coils, it generates an induction electromotive force to further create an eddy current I_e in the processed workpiece. The eddy current flows on sections of the workpiece and causes a loss of $I_e^2 R$. This eddy current loss generates heat on the heated workpiece in a manner similar to the above hysteresis loss. Hence, the eddy loss is also called Joule loss. The ratio of hysteresis loss to eddy current loss varies with the current frequency. A higher frequency during

heating corresponds to a higher proportion of eddy current loss. The eddy current loss is given by¹⁰

$$P_e = K_e(B_m \times f \times t)^2, \tag{2}$$

where P_e is the eddy current loss per kilogram; K_e is the constant of proportionality for eddy current loss; f is the current frequency; B_m is the peak flux density, and t is the thickness of the processed workpiece.

The electromagnetic effect comprises the skin effect, the proximity effect and the boundary effect, which are given as follows.

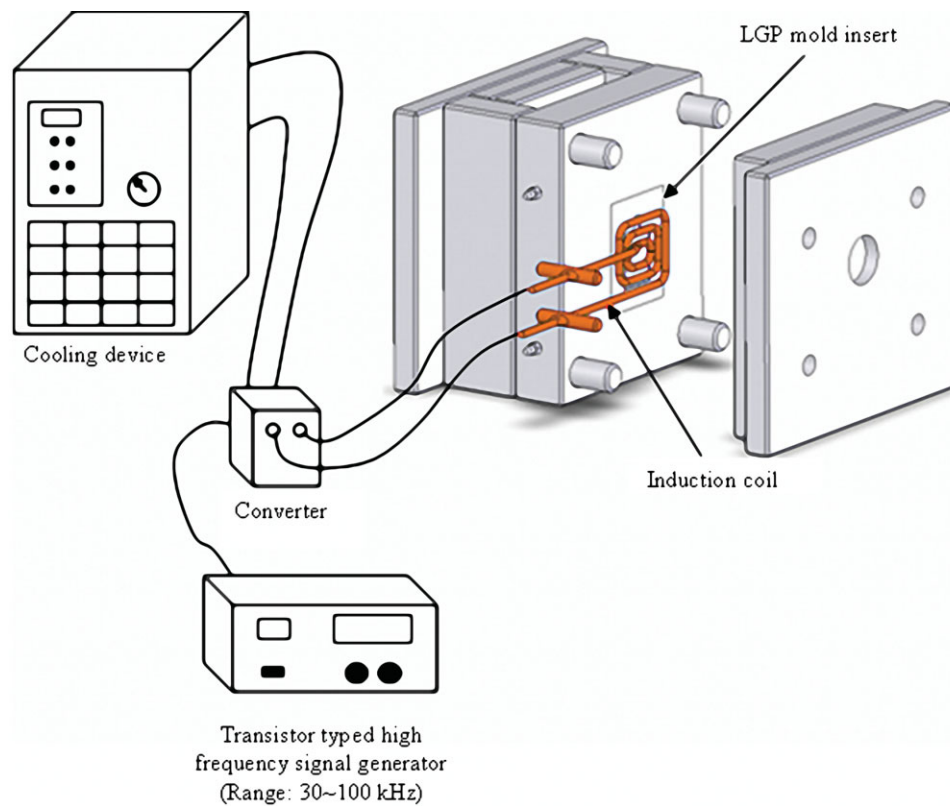


Figure 1 Experimental setup for rapid induction heating of LGP mold surface. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

Skin effect

When alternating current passes through a conductive wire, a magnetic field is generated both inside and outside the conductor. If the current varies, then the magnetic field in the conductor will vary, generating an induction electromotive force and, thus, a current in the conductor. The current in the conducting wire section is distributed as follows: the current near the center of the conducting wire is very small and that at the surface is larger, which effect is called the skin effect. In induction heating, when a

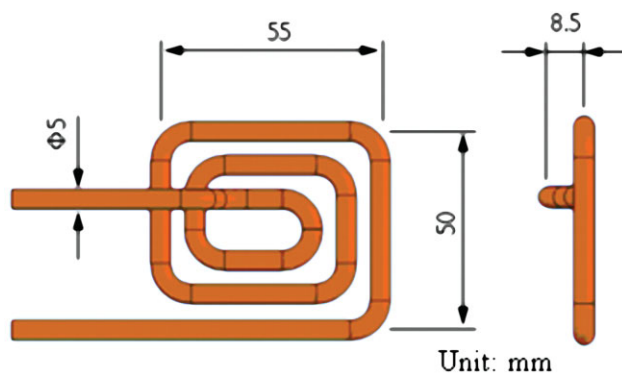


Figure 2 The induction coil's geometry and dimensions. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

high-frequency current passes through the coil, the induction eddy current in the processed workpiece is largest at its surface. The eddy current declines exponentially with distance from the surface.

Proximity effect

The proximity effect is the sudden interaction of magnetic fields that are generated because of flow of

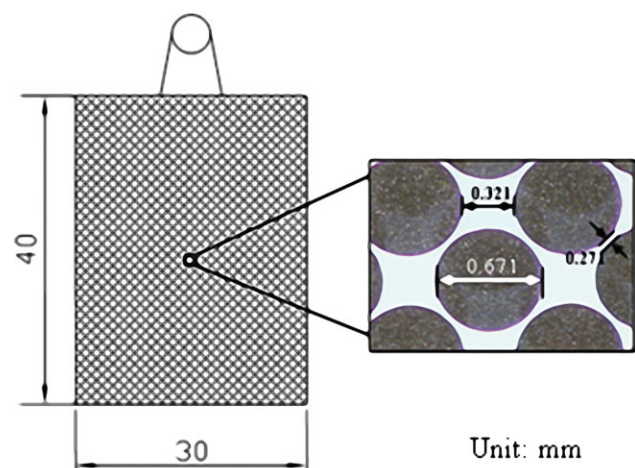


Figure 3 The microstructure of electroform stamper for LGP.

current in two adjacent charged conductors or coils, affecting the change of flux. Therefore, the path of induction current is similar to that of the inductance coil. A higher frequency or a smaller gap between the processed workpiece and the heating coil corresponds to a stronger proximity effect. The generated eddy current increases the flux that approaches the face, and causes the current to penetrate to a shallow depth. Thus, the degree of heating increases. In contrast, at a distance the flux is lower; the heating effect is weaker, and the skin effect influences only the distribution of the eddy current. However, the proximity effect changes only the magnitude of the eddy current.

The boundary effect is manifest in the action of the magnetic field in workpieces and the ends of coils. The combined effects of the workpiece and the ends of the coil cause power to be distributed along the length of the workpiece, affecting the distribution of temperature.

In conclusion, the factors that dominate the induction heating system are frequency of current and shape of the heating coil. Current frequency affects the depth of penetration. Although resistance and magnetic permeability are properties of the material, the current frequency can be selected. A higher frequency typically corresponds to shallower heat penetration. Heating coils are of many types, but only a few are extensively applied. Table II presents commonly used heating coils —grouped into single-turn and multi-turn types. The former are for small heating areas; the latter provide higher power, require a shorter heating time and are used for larger heating areas.¹¹

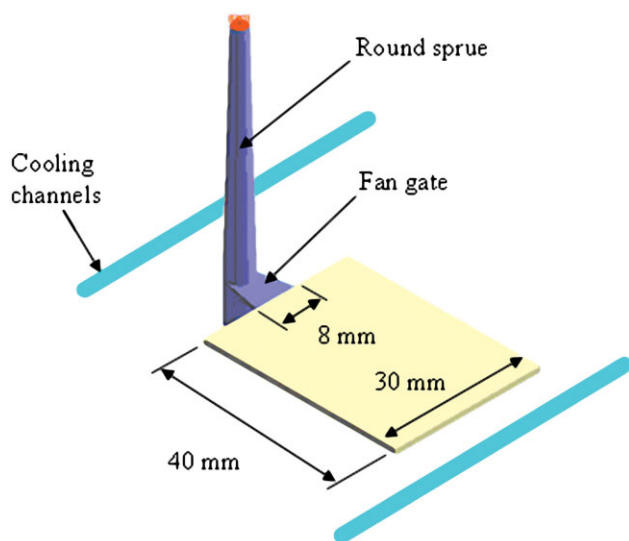


Figure 4 Geometry of LGP injection mold. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

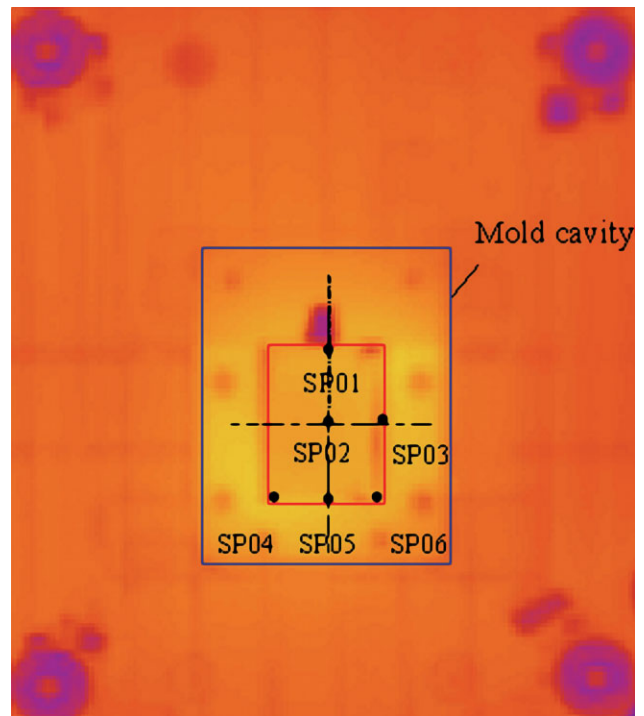


Figure 5 The measured locations of mold surface temperature in LGP. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

EXPERIMENT

In this work, LGP injection molding is conducted and quality control of the replication ability of LGP's

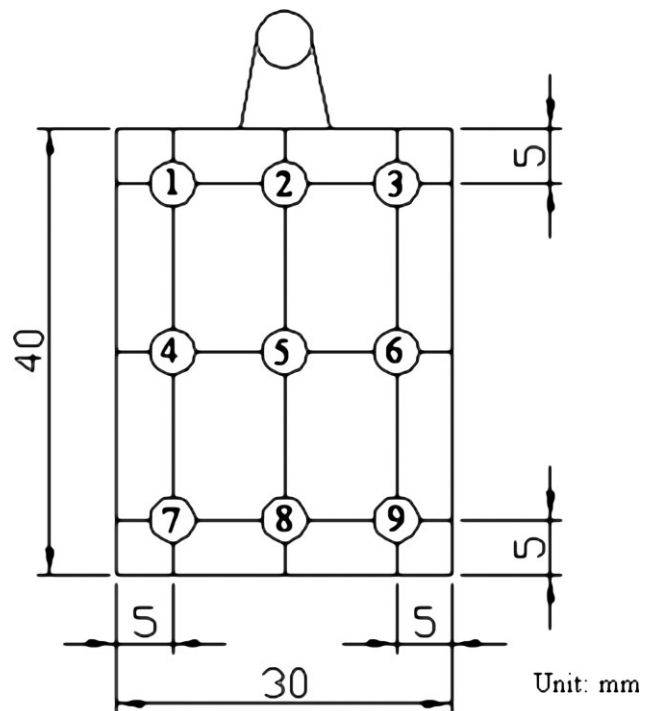


Figure 6 The measured locations of replicated heights in LGP.

TABLE III
Settings of Experimental Injection Molding parameters

	Without mold heating	Oil heating	Oil & induction heating	Induction heating		
Mold temp. (°C)	25	80	110	110	130	150
Cooling time (sec)	20	20	20	20	20	20
Injection speed (mm/s)	70	70	70	70	70	70
Injection pressure (kg/cm ²)	800	800	800	800	800	800
Injection stroke (mm)	13	13	13	13	13	13
Holding pressure (kg/cm ²)	600	600	600	600	600	600
Holding time (sec)	2	2	2	2	2	2
Plastics temp. (°C)	260	260	260	260	260	260
Back pressure (kg/cm ²)	80	80	80	80	80	80

microstructure is the objective. Figure 1 depicts the configuration of induction heating equipment to heat the LGP mold surface. The heating coil is a single turned copper wire that can be used to heat a small area. Figure 2 presents the geometry of an induction coil. The high-frequency induction heating equipment in the experiment is equipped with a transistor-type high frequency oscillator, a high-frequency output transformer, an induction coil and a water cooling system, with an input power of 25kW and an output oscillation frequency of 30–100 kHz.

As presented in Figure 3, the 2-inch molded part is a flat LGP of dimensions 4 mm long × 30 mm wide × and 1 mm thick. The microstructure is cylindrical (with diameter 671 μm and height 22.6 μm). This singular cavity mold as shown in Figure 4 is designed with a fan-shaped gate with a width of 8 mm and a thickness of 0.7 mm. The cooling system has two inlets and two outlets. A Japanese FANUC α-30i all electric injection molding machine is used to perform precise molding. The plastic used in this experiment was PMMA (Kurarary GH-1000S; Glass transition temperature: 104°C). In the experiment, a thermometer and an infrared ray thermal imaging system (ThermoVision A20M; Precision: within 100 ± 2°C; above 100 ± 2°C of the reading) are used to measure the temperature to record the variation in mold temperature because of induction heating. Thus, thermal images are captured to elucidate further the effect of current, frequency and duration of induction heating on the mold surface. Figure 5 displays the positions on the mold surface where the temperature was measured.

Under each molding condition, 10 pieces of LGP are used as samples. After the samples are cooled for 24 h, the size of LGP's microstructure is measured using a surface profiler. Figure 6 presents the measured locations of replicated heights in LGP, in which measurements are made over 1.5 mm at each gage point, and the height of the microstructure is represented as an average.

The designed experiments on 2-inch LGP have three stages. In the first stage, the induction heating

capacity is measured, and uniformity of heating is considered. In the second stage, the effects of the temperature of the unheated mold, oil heating, a combination of oil heating and induction heating, and pure induction heating, on the replication ability of LGP's microstructure are discussed. In particular, before injection and filling, the workpiece is heated to a temperature above the plastic glass transition temperature; during cooling, after filling, the temperature is reduced to below the plastic glass transition temperature. Table III presents the settings of experimental injection molding parameters conducted in this study to observe the variation in the replication ability of LGP's microstructure. In the final stage, only induction heating is used to control temperature and whether a higher mold temperature yields a greater replication ability of the microstructure is considered.

RESULTS AND DISCUSSION

Stage 1: Verification of inductive heating capacity

To verify the heating capacity of the induction heating equipment, the mold was initially heated to 80°C by oil heating and then the LGP mold surface was heated to above the glass transition temperature by induction heating. The induction heating parameters were set to current at 5.8 Ampere (A) and oscillation frequency at 40.21 kHz. The heating distance between the induction coil and the mold surface was 2 mm, as detailed in Table IV. Taking an induction heating time of 4 s as an example, the average temperature of the mold surface was 106.7°C—higher

TABLE IV
Induction Heating Device Settings for Various Temperatures

	80 ~ 110°C	130°C	150°C
Target	80 ~ 110°C	130°C	150°C
Current	5.8 A	5.8 A	5.8 A
Frequency	40.21 kHz	40.21 kHz	40.21 kHz
Heating distance	2 mm	2 mm	2 mm
Heating time	4 sec	3 sec	4 sec

TABLE V
Measured Temperatures that Following Induction Heating of LGP Mold Surface (unit: °C)

No.	SP01	SP02	SP03	SP04	SP05	SP06	Average	SD
1	109.0	101.3	109.7	113.2	113.2	114.0	110.1	4.8
2	100.7	103.4	110.6	111.1	107.8	105.9	106.6	4.1
3	102.8	106.5	112.4	116.1	109.9	108.2	109.3	4.6
4	103.8	104.7	110.1	111.4	108.0	106.6	107.4	3.0
5	103.9	104.1	110.6	109.2	108.2	108.6	107.4	2.8
6	108.0	110.0	118.3	114.0	109.9	106.7	111.2	4.3
7	100.9	107.6	111.4	111.5	107.5	105.3	107.4	4.0
8	109.0	108.0	115.3	114.1	111.3	108.5	111.0	3.1
9	107.8	109.3	114.2	114.8	110.4	105.0	110.3	3.8
10	113.0	112.4	116.3	115.6	110.3	104.7	112.1	4.2
Average	105.9	106.7	112.9	113.1	109.7	107.4	109.3	
SD	4.1	3.4	3.0	2.2	1.8	2.7		

than the glass transition temperature, 104°C, and able to support injection molding. Whether the mold surface temperature established by induction heating is uniform has been investigated. Table V presents the experimental results, in which the mold surface temperature exceeds the PMMA polymer's glass transition temperature of 104°C. However, the uniformity of the temperature obtained by induction heating is 6.7 ~ 12.7°C.

In the experiment, an induction controller with the coil is used in heating the LGP mold surface. Whether the mold surface temperature due only to induction heating reaches the expected value and whether the heating rate is high under such circumstances, as in a "rapid temperature rise", are discussed. The heating parameters of induction heating equipments are current at 5.8 A, frequency at 40.21 kHz, distance between induction coil and mold face being 2 mm, heating times being 3 and 4 s respectively, as indicated in Table IV. The temperatures of the mold are 130 and 150°C, respectively. To ensure that the set mold surface temperature is as required of induction heating, the cyclic change of temperature is measured using an infrared thermal imaging system. Figure 7 shows that control of LGP mold surface temperature profiles with respect to 3- and 4-s induction heating conducted in this experiment is reliable.

Stage 2: Effect of mold surface temperature on replication of microstructure of LGP by non-heating, oil heating, induction heating, oil heating, and pure induction heating

Table III presents the molding parameters used in the experiment, and Table VI presents the replication ability and temperature of the microstructure of the finished products. The experimental results demonstrate that replication ability of the microstructure increases with the temperature of the heated mold. In particular, when mold surface temperature exceeds plastic glass transition temperature, the fill-

ing efficiency is significant. Experimentally comparing the effects of various heating methods on the replication ability of LGP's microstructure indicates that the combination of an induction heating system and oil heater yields the best replication ability (when firstly, the oil heater is used to heat the mold to 80°C and then the induction heating system is used to heat it to 110°C). This combination yields a replication rate of 95.0%, which is 6.0% higher than that obtained using the oil heater to heat to 80°C, and 7.8% higher than that of an unheated mold. Consequently, induction heating may effectively increase the filling efficiency of LGP's microstructure. Figure 8 shows SEM photographs of micro-injection molded LGP generated with the induction heating.

Stage 3: Effect of induction heating to high temperature on replication ability of LGP's microstructure

Table III presents the injection molding parameters used in the experiment and Table VII the replication ability of LGP's microstructure. In the experiment, the mold is heated and the effect of temperature on the replication ability of LGP's microstructure is examined. When the mold is induction-heated to

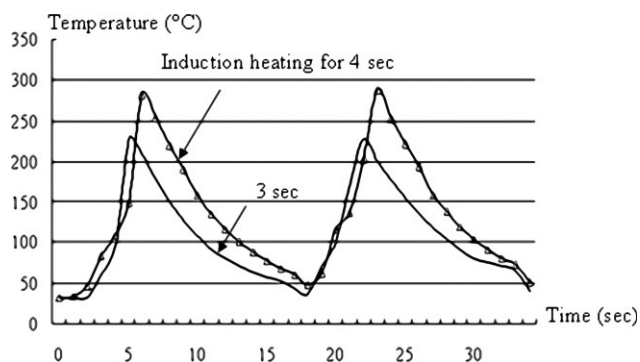


Figure 7 The mold temperature profiles with induction heating for 3 and 4 s, respectively.

TABLE VI
Replication Heights of LGP's Microstructure for Various Heating Settings

Heating type Position	Without mold heating			Oil heating to 80°C			Oil heating to 80°C followed with induction heating to 110°C		
	Height (μm)			Height (μm)			Height (μm)		
	Average	SD	%	Average	SD	%	Average	SD	%
1	21.4	1.5	89.0	21.8	1.5	90.6	23.1	0.6	96.3
2	21.6	1.1	89.5	21.9	1.2	90.9	23.1	0.7	95.9
3	19.0	0.4	89.3	19.1	0.7	89.4	20.9	0.4	98.1
4	19.1	0.5	86.9	19.0	0.7	86.6	20.2	0.6	91.8
5	19.5	0.6	82.3	20.8	1.1	87.9	22.6	0.9	95.4
6	20.0	0.3	89.1	20.4	0.6	90.5	21.3	0.8	94.7
7	20.1	1.2	90.6	20.8	0.4	93.8	21.1	0.6	95.0
8	19.5	1.2	85.4	20.1	1.2	88.3	21.9	0.7	95.9
9	19.8	0.8	87.0	19.8	0.9	86.9	20.9	0.7	91.5
Average	20.0	0.8	87.2	20.4	0.9	89.0	21.7	0.7	95.0
Range	2.6		8.3	2.9		7.2	2.9		6.6

110°C, the replication ratio is 92.7%; when it is induction-heated to 130°C, the replication ratio is 92.8%; and when it is heated to 150°C, the replication ratio is 92.9%. The experimental results show that when the mold temperature exceeds the plastic glass transition temperature, increasing the mold temperature markedly improves the replication ability of the microstructure.

Injection molding results in centralization or non-uniformity of interior stress in plastics, affecting the light transmittance, brilliance and uniformity of the light-guided plate. Generally, during injection molding, residual stress in plastic part is produced by the high shear stress that is caused by the flow of melt plastics in the filling stage or the residual stress and heat stress that results from the high dwell pressure in the post-filling stage. The heat stress normally results from uneven cooling and temperature distribution. In the experiment, induction heating is utilized to heat the mold surface rapidly, affecting the mold temperature and its effect on the residual stress in LGP, which are both discussed. Residual stress in a transparent injected finished LGP can be denoted by a birefringence value, which is calculated based on the photoelastic principle. The Appendix presents relevant details. Table VIII reveals that using induction heating of a mold to a high temperature heating causes negligible residual stress in LGP (which is directly proportional to birefringence value). A small shear stress is produced when the melt plastics is injected into the mold at high temperature and prolongs the production of the solidified layer on the mold surface.

CONCLUSIONS

A high mold temperature has been proven to significantly improve the level of filling with melt plastics

in microinjection molding. Accordingly, this study introduces induction heating of a mold surface and applies it to LGP's microstructure. The experimental results prove that induction heating is more economically effective than traditional oil heating and improves molding quality. The following conclusions are drawn.

1. The experimental results show that the LGP mold surface heated by the induction heating system to obtain the desired molding temperature requires only 3 s, indicating the advantage of efficiency. However, the experimental results demonstrate that the temperature uniformity achieved by the induction heating system is only 6.7 ~ 12.7°C.

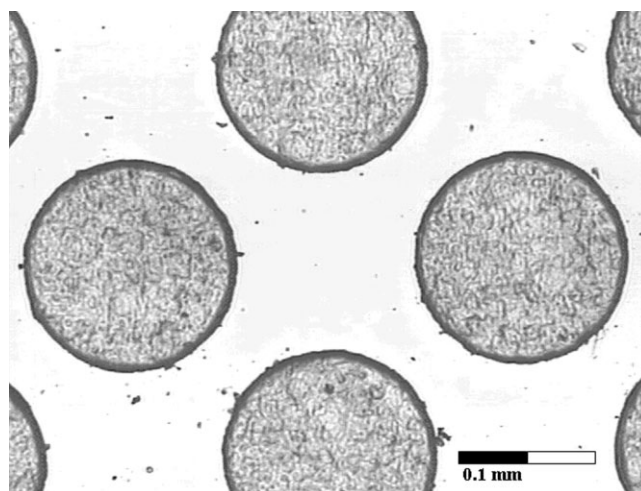


Figure 8 The SEM photographs of micro-injection molded 2-inch LGP.

TABLE VII
Replication Heights of LGP's Microstructure Achieved by Induction Heating with Various Set of Mold Temperature

Heating type Position	Induction heating to 110°C			Induction heating to 130°C			Induction heating to 150°C		
	Height (μm)			Height (μm)			Height (μm)		
	Average	SD	%	Average	SD	%	Average	SD	%
1	22.4	0.5	93.1	22.4	0.7	93.3	22.3	0.5	93.0
2	22.2	1.3	92.2	22.0	1.0	91.2	22.4	1.4	93.0
3	20.2	0.5	94.7	20.1	0.5	94.4	20.1	0.8	94.4
4	19.9	0.5	90.4	19.4	0.8	88.4	19.7	0.4	89.7
5	22.2	1.1	93.5	22.2	1.0	93.6	22.2	1.3	93.8
6	20.8	1.0	92.4	20.7	1.0	92.2	20.6	0.8	91.4
7	21.1	0.5	95.0	21.1	0.5	95.2	21.1	0.3	95.2
8	21.3	0.9	93.3	21.6	0.7	94.5	21.9	0.7	96.1
9	20.5	0.8	90.0	21.1	1.3	92.5	20.5	0.8	89.8
Average	21.2		92.7	21.2		92.8	21.2		92.9
Range	2.5		5.0	3.0		6.8	2.7		6.4

TABLE VIII
Distribution of Residual Stress in Injection Molded LGP

Heating type	Without mold heating	Oil heating to 80°C	Oil heating to 80°C followed with induction heating to 110°C	Induction heating to 110°C	Induction heating to 130°C	Induction heating to 150°C
Maximal value of birefringence ($\times 10^7$)	1.03	1.02	0.93	0.93	0.89	0.88

- The experimental results indicate that a high mold temperature is conducive to the molding of microstructure, and that the effect is the optimal when the mold temperature exceeds the plastic glass transition temperature. The experiment indicates that the use of a combination of an induction heating system and oil heater yields the best replication ability (by firstly heating using the oil heater to 80°C and then heating by induction to 110°C). This combination yields a replication ratio of 95.0%, which is 6.0% higher than obtained using an oil heater to heat to 80°C and 7.8% higher than of an unheated mold. Therefore, induction heating effectively improves filling efficiency of LGP's microstructure.
- The experimental results also reveal that when the mold temperature exceeds the plastic glass transition temperature, increasing the mold temperature will not significantly improve the replication ability of LGP's microstructure. In addition, the photoelastic experiment demonstrated that induction heating technology may produce a high mold temperature and will not cause much residual stress of the LGP.

The authors would like to appreciate Ted Knoy for his editorial assistance.

APPENDIX

Photo-elasticity analysis is an optical full-range stress measurement method. Accordingly, transparent birefringence photo-elastic materials are used to make a model that is similar in shape to the actual object and to load it to an extent that is similar to that in a practical situation. When light is incident, birefringence occurs temporarily, observed as a photo-elastic effect. If the object is placed in a polarized light field, the interference caused by the difference between the phases of the polarized light because of stress, produces dark and light stripes, which reveal the stress distribution among points inside the model. On the basis of photo-elastic principle, the stresses at all positions inside the model may be determined.

In the image-processing system, the light intensity is called the gray level and is related to actual light intensity as follows;

$$Z = BI^\gamma, \quad (A1)$$

where, Z is the gray level at the point; B is a constant of proportionality; I is the light intensity at the point, and γ is the slope of the sensitivity curve of the CCD camera.

The dark-field device is associated with the corresponding gray level, given by eq. (A2), in the image-

processing system, based on the light intensity equation, eq. (A3), based on traditional photo-elastic principles:

$$Z = B[I_0 \sin^2(N\pi)]^r, \quad (\text{A2})$$

$$I = I_0 \sin^2 \frac{\delta}{2} = I_0 \sin^2(N\pi). \quad (\text{A3})$$

When the light intensity is maximal, if the largest gray level is Z_{\max} , then,

$$\frac{Z}{Z_{\max}} = \left(\frac{I}{I_{\max}}\right)^r = \left[\frac{\sin(N\pi)}{\sin(N_{\max}\pi)}\right]^{2r}, \quad (\text{A4})$$

where, N_{\max} is the largest number of stripe value corresponding to Z_{\max} . In the half-sequence photo-elastic method, $N_{\max} = 0.5$, $\sin(N_{\max}\pi) = 1$, and,

$$\frac{Z}{Z_{\max}} = [\sin(N\pi)]^{2r}, \quad (\text{A5})$$

$$N = \frac{1}{\pi} \sin^{-1} \left[\left(\frac{Z}{Z_{\max}} \right)^{\frac{1}{2r}} \right], \quad (\text{A6})$$

where, N is the number of stripes, and Z_{\max} is the gray level that corresponding to the highest light intensity. The γ value must always be checked before photo-elastic data can be obtained. In the correction, the Tardy compensation method is adopted to find various groups of N and Z values that correspond to each other and are then converted into X and Y values. Then, the least squared error method is applied to obtain the optimal correction factors, A and γ .

Accordingly, the gray level and the corresponding stripe value may be obtained using the following equation.

$$N = \frac{1}{\pi} \sin^{-1} \left[\left(\frac{Z}{AZ_{\max}} \right)^{\frac{1}{2r}} \right]. \quad (\text{A7})$$

In the experiment, digital photo-elastic is obtained for a stripe value N at each point by using the following equation.

$$\Delta n = \frac{N\lambda}{d}. \quad (\text{A8})$$

Then, difference among the birefringence values at all points may be determined.¹²

References

1. Yao, D.; Kim, B. *J Mater Process Technol* 2002, 6, 11.
2. Yoshii, M.; Kuramoto, H.; Ochiai, Y. *Polym Eng Sci* 1998, 38, 1587.
3. Kim, D. H.; Kang, M. H.; Chun, Y. H. *J Mater Process Technol* 2001, 5, 229.
4. Yao, D.; Kim, B. *Polym Eng Sci* 2002, 42, 2471.
5. Chang, P. C.; Hwang, S. J. *J Appl Polym Sci* 2006, 102, 3704.
6. Chen, S. C.; Jong, W. R.; Chang, Y. J.; Chang, J. A.; Cin, J. C. *J Micromech Microeng* 2006, 16, 1783.
7. Yao, D.; Kimerling, T. E.; Kim, B. *Polym Eng Sci* 2006, 46, 938.
8. Davies, E. J.; Simpson, P. G. *Induction Heating Handbook*; McGraw-Hill Book Company Ltd.: London, 1979.
9. Robert, P. *Electrical and Magnetic Properties of Materials*; Artech House: London, 1988.
10. Garcia, J. R.; Burdío, J. M.; Martínez, A.; Sancho, J. In *Proceedings of the IEEE APEC Records*; IEEE: New York, 1994; p 302.
11. Sue, J. S. Master Thesis, Chung-Yuan Christian University, 2003.
12. Chen, Y. C.; Chen, C. H.; Chen, S. C. *Polym Int* 1996, 40, 846.